J. Fluid Mech. (1972), *vol.* 53, *part* 3, *pp.* 575–586
Printed in Great Britain

The influence of drag-reducing polymers on turbulence in the viscous sublayer

By GILEAD FORTUNA AND THOMAS J. HANRATTY
Department of Chemical Engineering, University of Illinois

(Received 24 March 1971 and in revised form 23 November 1971)

The influence of drag-reducing polymers on the time-averaged velocity gradient $\overline{S_x}$ and on the two components of the fluctuating velocity gradient at the wall, s_x and s_z, has been studied for turbulent flow in a 1 in. pipe. The time-averaged velocity gradient is related to the time-averaged wall stress by Newton's law of viscosity. The view is taken that a comparison of the turbulence structure with and without polymers should be made at the same $\overline{S_x}$. Therefore all turbulence measurements have been normalized with wall parameters. On this basis we find that the changes in the intensity of s_x and in the shape of the spectral density functions for s_x and s_z are not as striking as the decrease in pressure drop caused by the addition of a polymer. The root-mean-square value of s_z shows a significant decrease with an increase in drag reduction but the most spectacular change in turbulence structure seems to be associated with the transverse correlation coefficient. We conclude that the increase in drag reduction is accompanied by an increase of the size of longitudinally oriented eddies in the viscous sublayer. One possible explanation for this increase in size is that the influence of the polymer additives is such as to increase the viscous resistance in the transverse direction more than in the direction of mean flow.

1. Introduction

At present there is evidence which suggests that the reduction of turbulent drag on a solid surface by the addition of polymer to a solvent is associated with changes in the turbulence in regions very close to the surface. Wells & Spangler (1967) injected dilute solutions of drag-reducing fluids both into the turbulent core and into the wall layer and found that the local pressure gradient is reduced only if the polymer is in the wall region. Meyer (1966) showed that measurements of average velocities of polymer solutions are correlated with the equation

$$\overline{U}^+ = A \ln y^+ + B, \tag{1}$$

where the superscript indicates that the average velocity $\overline{U}$ and the distance from the wall y have been made dimensionless with wall parameters, the friction velocity u^* and the kinematic viscosity ν. The constant A has the same value as that measured for the pure solvent; however, B is larger for the solution than for the solvent and increases as drag reduction increases. This increase in B is interpreted as a thickening of the viscous sublayer.

This paper presents the results of experiments in which we employ electro-chemical techniques, developed by Reiss & Hanratty (1963), to determine changes of turbulence structure caused by the addition of polymers to an aqueous solution. An electro-chemical reaction whose rate is controlled by mass transfer is carried out on a small electrode mounted flush with the wall. The velocity gradient at the wall is calculated from the current flowing in the electro-chemical circuit. This type of measuring probe has the advantage of offering no obstruction to the flow.

2. Description of the experiments

The electro-chemical techniques used in this research have been described in previous papers, so only those problems associated with their application to polymer solutions will be discussed here.

Mitchell & Hanratty (1966) measured the x component of the velocity gradient, S_x, in a 1 in. pipe by using rectangular electrodes with their long sides perpendicular to the direction of mean flow. Sirkar & Hanratty (1970*b*) used two rectangular electrodes in a \/ arrangement to measure the transverse component of the velocity gradient, s_z. In recent experiments problems associated with spatial averaging over the electrode surface encountered by Mitchell & Hanratty have been avoided by using a larger pipe or smaller test electrodes. Thus Sirkar (1969) used rectangular electrodes with a 7·615 in. pipe and Fortuna (1971), circular electrodes of diameter 0·004 in. with a 1 in. pipe.

The electrolyte consisted of potassium ferrocyanide (0·01 N), potassium ferricyanide (0·01 N) and a large excess of sodium hydroxide (0·1 N) dissolved in water. The effects of three polymers were investigated: Polyox WSR 301, supplied by Union Carbide; Polyhall 295, supplied by Stein & Hall; and Separan AP-30, supplied by Dow Chemical. Experimental difficulties were encountered since the polymers tended to degrade faster in the electrolyte solution than in pure water and since drag reduction occurred at high enough friction velocities for some of the effects of polymer addition to be obscured if the frequency response of the probes was not considered. This necessitated an extension of the analysis of Mitchell & Hanratty (1966) to include higher frequencies (Fortuna & Hanratty 1971*b*).

Our initial experiments were done in a Couette system with an outer diameter of 5·025 in. and an inner diameter of 4·842 in. Fresh fluid was fed in continuously in the axial direction at a very slow rate in order to control degradation of the polymer and the outer cylinder was rotated at various velocities. The torque transmitted to the stationary inner cylinder was measured with a strain gauge. This system was ideal for preliminary experimentation, since it needed small quantities of fluid and since there was no concern regarding polymer degradation. Final experiments were performed in our flow loop, which has a 1 in. diameter pipe as a test section (Mitchell & Hanratty 1966). The length of the runs in the 1 in. pipe was limited because of polymer degradation. However, this difficulty was lessened to some extent by using low concentrations of sodium hydroxide and by installing a one-stage screw-type Moyno pump with a stainless-steel rotor and rubber stator.

The diffusion coefficient of ferricyanide ions in polymer solutions was needed to calculate the velocity gradient from measured mass-transfer coefficients. This was determined by operating the Couette system with a laminar flow. Since $\overline{S_x}$ is known, D can be calculated by the following mass-balance equation for the electrode:

$$\bar{K} = \frac{3}{9^{\frac{1}{3}}2\Gamma(\frac{4}{3})}\left(\frac{D^2\overline{S_x}}{L}\right)^{\frac{1}{3}}, \tag{2}$$

where $\bar{K}$ is the time-averaged mass-transfer coefficient; D, the diffusion coefficient; L, the effective length of the electrode and $\overline{S_x}$, the time averaged velocity gradient at the wall. The addition of small amounts of polymer caused only slight changes in D. Some of these results have been presented in a previous paper (Fortuna & Hanratty 1971*a*).

The viscosities of the polymer solutions were determined by a Cannon–Fenske size 25 capillary viscometer. This gave a velocity gradient at the wall of about $1500\,\mathrm{s}^{-1}$, which is approximately equal to or less than the values of $\overline{S_x}$ measured under drag-reducing conditions. Viscosities of the polymer solutions used in this research, as measured with the capillary viscometer, never exceeded the value for the solvent by more than 10 per cent. The presence of hydroxyl ions apparently affected the state of the polymer molecules since solutions in distilled water showed higher viscosities.

3. Experimental results

Measurements were obtained of the average velocity gradient at the wall ($\overline{S_x}$), the mean square of the x component ($\overline{s_x^2}$) and of the z component ($\overline{s_z^2}$) of the fluctuations in the velocity gradient at the wall, the spectral density functions for s_x and s_z, and the x and z correlation functions for s_x.

3.1. *The average velocity gradient*

The average velocity gradient was measured both in the Couette system and in the pipe. The wall shear stress was found to be related to the time-averaged velocity gradient by the Newtonian relation.

Friction factors calculated from torque measurements on the inner cylinder of the Couette apparatus are shown in figure 1 for the electrolyte solution before and after 100 p.p.m. of Polyox WSR 301 had been dissolved in it.

$$f_D = \langle\tau\rangle/\tfrac{1}{2}\rho U^2, \tag{3}$$

where $\langle\tau\rangle$ is the average stress on the wall and U is the velocity of the rotating outer cylinder. Time-averaged velocity gradients were also measured using circular electrodes of diameter 0·020 in. embedded in the wall of the inner cylinder close to its centre. Friction factors calculated from these measurements,

$$f_v = \mu\overline{S_x}/\tfrac{1}{2}\rho U^2, \tag{4}$$

are shown in figure 2. For the Newtonian fluid, f_v is 20–30 % lower than f_D because of end effects in the torque measurements. The important finding is

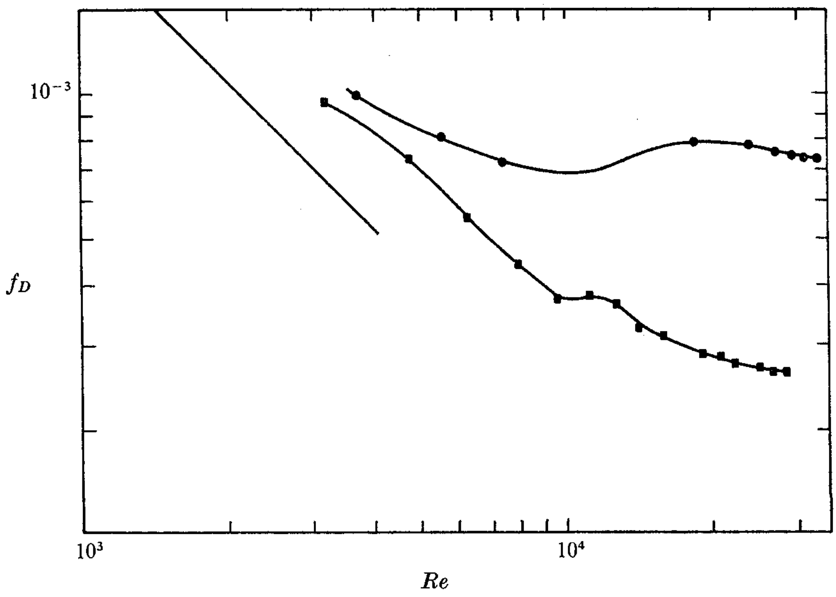

FIGURE 1. Torque measurements in the Couette system. $Re = Ud/\nu$, $f_D = \langle\tau\rangle/\tfrac{1}{2}\rho U^2$. ●, Newtonian solvent; ■, 100 p.p.m. Polyox WSR 301.

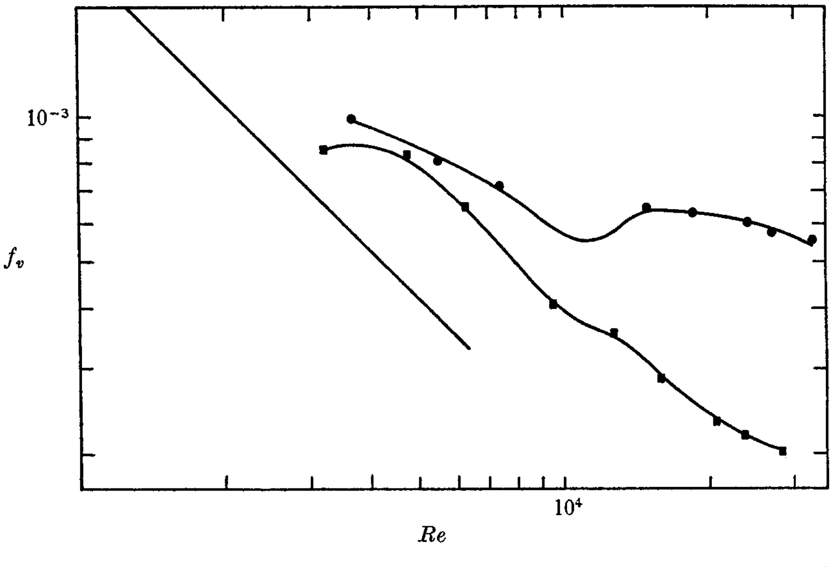

FIGURE 2. Velocity gradient measurements in the Couette system. $Re = Ud/\nu$, $f_v = \mu\overline{S_x}/\tfrac{1}{2}\rho U^2$. ●, Newtonian solvent; ■, 100 p.p.m. Polyox WSR 301.

that at large Reynolds numbers where the turbulence appears fully developed ($Re = 30\,000$) the addition of the polymer causes the same percentage decrease in f_v and in f_D: 65 %.

The complete friction factor versus Reynolds number relations for polymer solutions were not measured in the flow loop because degradation limited the duration of an experiment. However, measurements taken at specific flow rates

Polymer	Concentration (p.p.m.)	Reynolds number	Reduction of pressure drop (%)	Reduction of velocity gradient (%)
Separan AP-30	160	28 000	45·9	44·5
	—	37 000	54·0	54·0
	—	117 000	76·0	71·5
Polyhall 295	110	102 000	51·8	52·5

TABLE 1. Reduction of drag and velocity gradient for flow in a 1 in. pipe

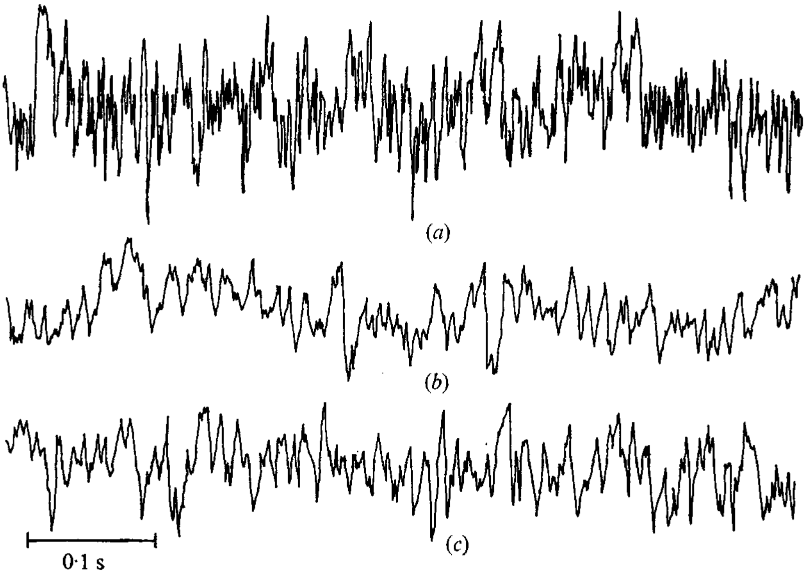

FIGURE 3. Fluctuation of axial velocity gradient. (*a*) Newtonian fluid, $Re = 105\,000$, $u^* = 18{\cdot}3$ cm/s. (*b*) 200 p.p.m. AP-30, $Re = 115\,000$, $u^* = 9{\cdot}8$ cm/s. (*c*) Newtonian fluid, $Re = 52\,000$, $u^* = 9{\cdot}8$ cm/s.

with different fresh polymer solutions show that the velocity gradient is reduced by the same percentage as the pressure drop, or drag. Some results are presented in table 1.

3.2. *Intensity of the fluctuations in the velocity gradient*

The effect of drag-reducing polymers on the axial component of the fluctuating velocity gradient at the wall, s_x, is illustrated in figure 3. A comparison of the recorded signal for the solvent at a Reynolds number of 105 000 with the signal for a solution with 200 p.p.m. of Separan AP-30 at approximately the same flow rate shows that the frequency and the amplitude of the fluctuations are greatly reduced by the addition of polymer. As can be seen from figures 3(*b*) and (*c*), the signal for the polymer solution is much closer to that for the solvent if the comparison is made at the same friction velocity or the same pressure drop.

For a Newtonian fluid the root-mean-square values of s_x and s_z, normalized with respect to $\overline{S_x}$, are approximately constant over the range of Reynolds

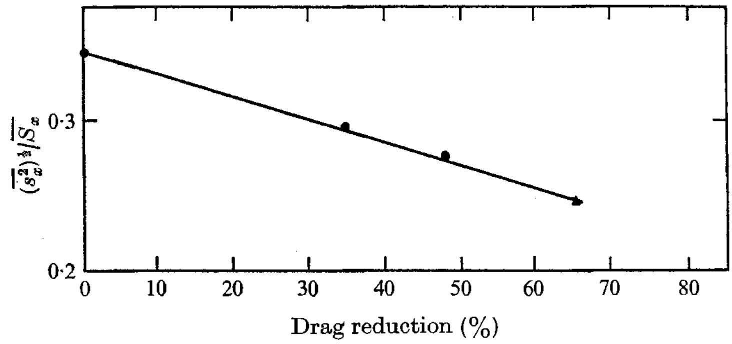

FIGURE 4. Relative intensity of the axial component of the fluctuating velocity gradient for flow of solutions of Separan AP-30 in a 1 in. pipe. ●, $Re = 27\,600$, ▲, $Re = 57\,600$.

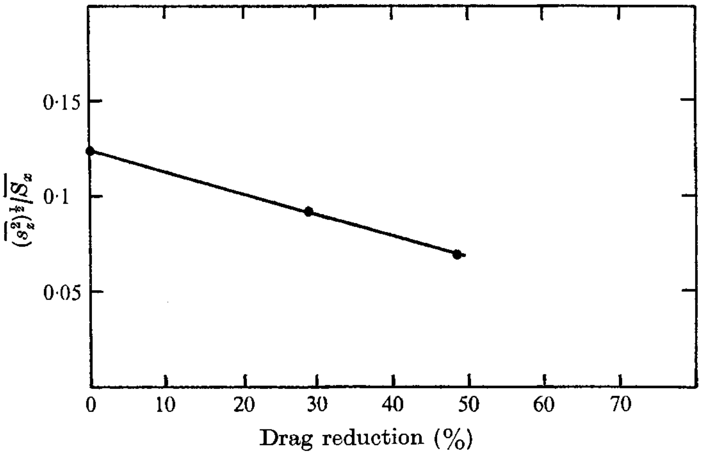

FIGURE 5. Relative intensity of the transverse component of the fluctuating velocity gradient for flow of solutions of Separan AP-30 in a 1 in. pipe. $Re = 31\,000$.

number covered in this investigation. Mitchell & Hanratty (1966) report a value for $(\overline{s_x^2})^{\frac{1}{2}}/\overline{S_x}$ of 0·32 and Sirkar & Hanratty (1970*b*), a value for $(\overline{s_z^2})^{\frac{1}{2}}/\overline{S_x}$ of 0·09–0·1 for a Newtonian fluid. More recent investigations by Sirkar (1969), Fortuna (1971) and by Eckelman (1971) suggest that $(\overline{s_x^2})^{\frac{1}{2}}/\overline{S_x} = 0{\cdot}35$–$0{\cdot}36$ and that $(\overline{s_z^2})^{\frac{1}{2}}/\overline{S_x} = 0{\cdot}11$–$0{\cdot}12$ for a Newtonian fluid. The presence of drag-reducing polymers causes a reduction both in $(\overline{s_x^2})^{\frac{1}{2}}/\overline{S_x}$ and in $(\overline{s_z^2})^{\frac{1}{2}}/\overline{S_x}$.

Values of $(\overline{s_x^2})^{\frac{1}{2}}/\overline{S_x}$ measured for solutions of Separan AP-30 with circular electrodes of 0·004 in. diameter in the 1 in. pipe are shown in figure 4. The percentage decrease in $(\overline{s_x^2})^{\frac{1}{2}}/\overline{S_x}$ observed at a fixed volumetric flow is not so great as the decrease in $(\overline{s_x^2})^{\frac{1}{2}}$ or the decrease in pressure drop. Values of $(\overline{s_z^2})^{\frac{1}{2}}/\overline{S_x}$ measured with two slant electrodes 0·002 × 0·020 in. are plotted in figure 5. It is noted that the addition of polymer causes a greater percentage decrease in $(\overline{s_z^2})^{\frac{1}{2}}/\overline{S_x}$ than in $(\overline{s_x^2})^{\frac{1}{2}}/\overline{S_x}$.

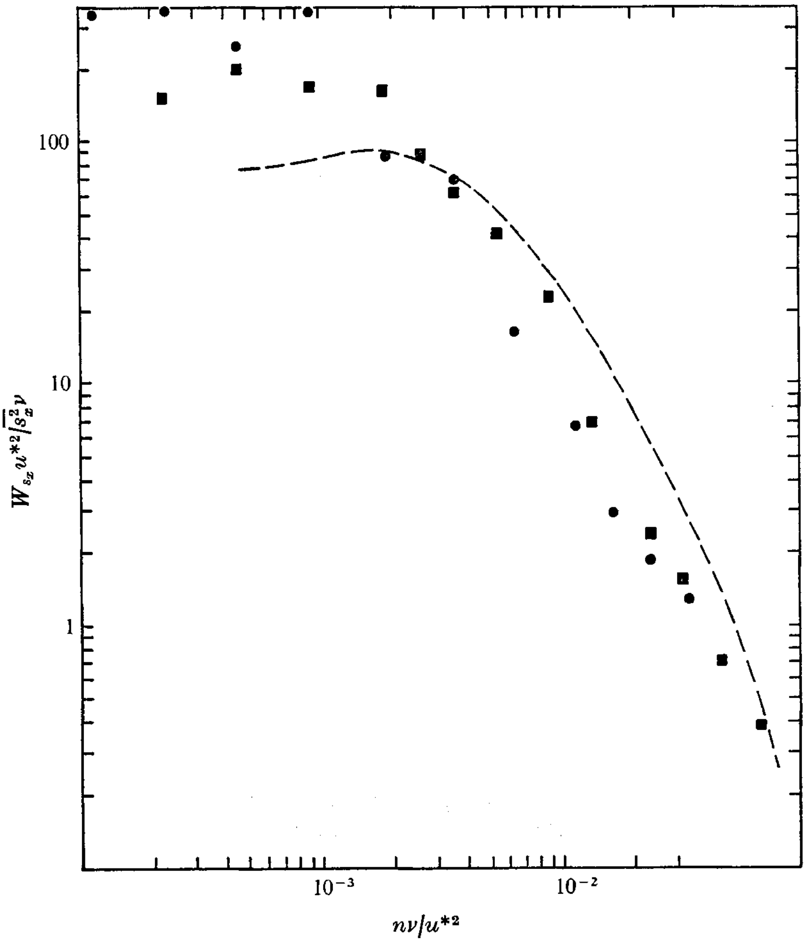

FIGURE 6. Spectral density function for s_x for solutions of Separan AP-30. ———, solvent data; ●, 34·6 % drag reduction, $Re = 27\,600$; ■, 64·5 % drag reduction, $Re = 57\,600$.

3.3. *Spectra*

Spectral density functions for s_x defined by the equation

$$\overline{s_x^2} = \int_0^\infty W_{s_x}\, dn, \tag{5}$$

where n is the frequency in cycles per second, are shown in figure 6 for turbulent flow of Separan AP-30 in a 1 in. pipe. The spectral density function for s_z is shown in figure 7. The relative contribution of the low frequency part of the spectra is increased with increasing drag reduction.

Of most interest is the finding that the spectral density functions for solutions of drag-reducing polymers are not dramatically different from those for Newtonian fluids when the variables are normalized with wall parameters.

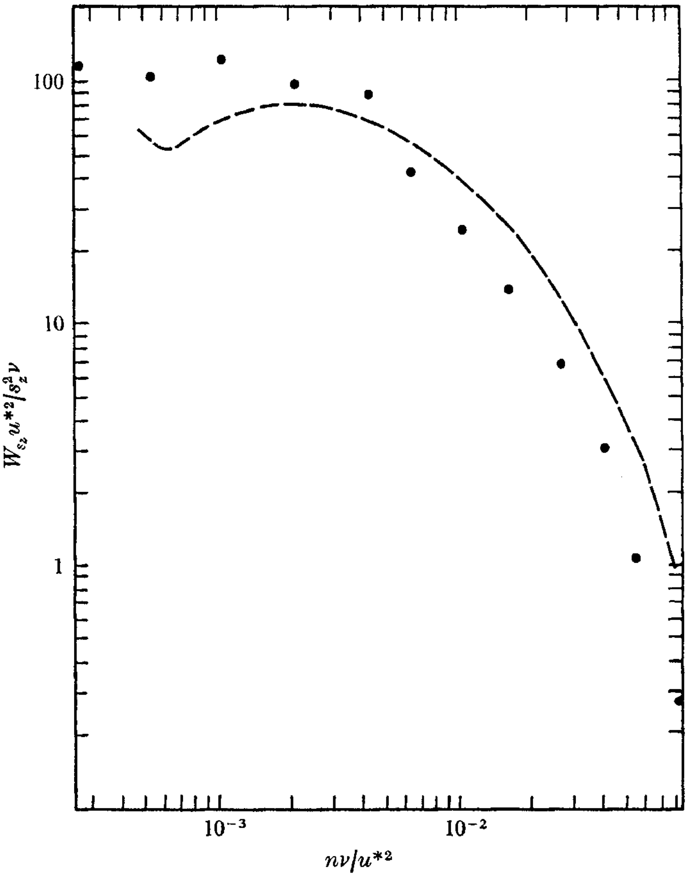

FIGURE 7. Spectral density function for s_z for solutions of Separan AP-30. ---, solvent data; ●, 48·5 % drag reduction, $Re = 31000$.

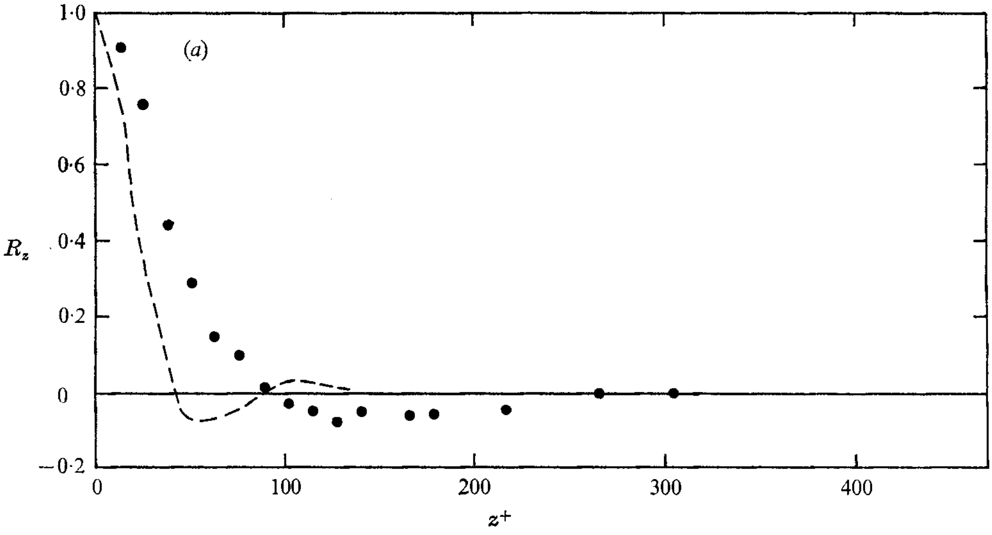

FIGURE 8 (a). For legend see facing page.

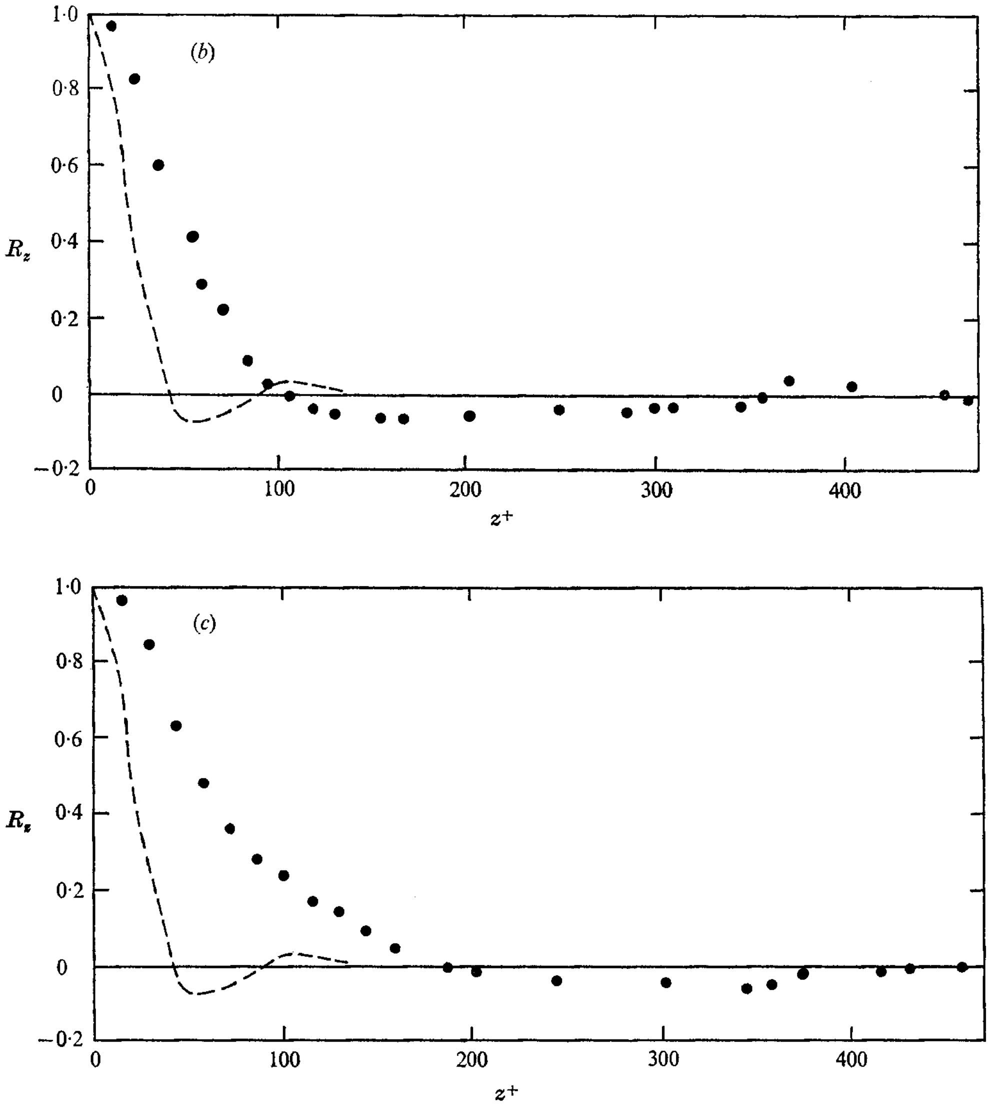

FIGURE 8. Spatial correlation coefficient measured in the z direction for a solution of Separan AP-30. ---, solvent. (*a*) ●, 27 % drag reduction, $Re = 27\,600$. (*b*) ●, 34·6 % drag reduction, $Re = 27\,600$. (*c*) ●, 49·3 % drag reduction, $Re = 39\,200$.

3.4. *Correlation coefficient*

Correlation coefficients of s_x measured in the transverse direction, R_z, are shown in figure 8 for solutions of Separan AP-30. A large increase in the scale in the z direction is caused by the addition of drag-reducing polymers. Negative correlations were measured at large distances with Newtonian fluids and with polymer solutions causing drag reductions of 27·0, 34·6 and 49·3 %. No negative correlations were detected with solutions having 65 % drag reduction, the maximum which could be achieved at a Reynolds number of 27 600 in the 1 in. pipe. Table 2 presents the scales calculated from the measurements of R_z and the distances corresponding to zero correlation, normalized with respect to wall parameters.

Correlations of s_x in the axial direction were measured only for extreme drag reduction and are shown in figure 9. The axial correlation coefficient is much larger than the Newtonian value for the same separation distance. This indicates that for polymer solutions the scale of s_x in the x direction is much larger than the scale in the z direction, as has been found for Newtonian fluids (Mitchell & Hanratty 1966).

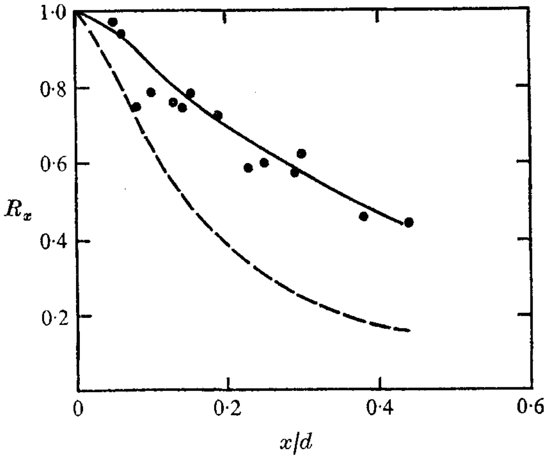

FIGURE 9. Spatial correlation coefficient measured in the x direction for a solution of Separan AP-30. ---, solvent; —●—, 65 % drag reduction, $Re = 95000$.

4. Interpretation

The viewpoint taken in this paper is that comparisons of turbulence measurements should be made at the same value $\overline{S_x}$ rather than at the same flow rate, in order to discover changes in the viscous sublayer caused by the addition of drag-reducing polymers. Therefore, all experimental variables characterizing the fluctuating flow have been normalized with respect to wall parameters. On this basis we find that the changes in the intensity of the axial velocity fluctuations and the changes in the shape of the spectral density functions are not as great as the decrease in the friction factor. The transverse velocity fluctuations show a significant decrease with increased drag reduction but the most spectacular change in turbulence structure seems to be associated with the transverse correlation coefficient.

Kline *et al.* (1967) and Bakewell & Lumley (1967) have suggested that turbulent flow in the viscous sublayer is dominated by a large eddy structure consisting of pairs of counter-rotating elongated eddies. The visual studies of Kline *et al.* (1967) suggest that the average distance between plumes emerging from the wall, normalized with respect to wall parameters, is $\lambda^+ \simeq 100$. The measurements of transverse correlations presented in this paper are consistent with such a model in that negative values are observed at large distances. If we take twice the

Drag reduction (%)	Scale Λ_z^+	z^+ at first zero of correlation	z^+ at second zero of correlation
0 (Newtonian)	17	40	90
27·0	31	95	265
34·6	40	105	355
49·3	62	185	430
64·5	114	720	—

TABLE 2. Transverse correlation distances

distance between the zeros of the correlation function as a measure of λ^+, we see from table 2 that the size of the large eddy structure estimated from correlation measurements for a Newtonian fluid is in close agreement with the values given by Kline *et al.* (1967). From these observations we associate an increase in drag reduction with an increase in the width of the longitudinally oriented eddies which have been reported to exist in the sublayer. From table 2, we see that a decrease in the pressure drop by a factor of two is associated with an approximately five-fold increase in λ^+.

Sirkar & Hanratty (1970*a*) have recently used a simplified model of the eddy structure proposed by Kline *et al.* (1967) and by Bakewell & Lumley (1967) to explain turbulent mass transfer to a wall at large Schmidt numbers. This simplified model can also be used to describe momentum transfer to a wall and implies that the changes in λ^+ suggested by the correlation measurements are sufficient to explain the observed drag reduction. The details of this analysis are contained in a thesis by Fortuna (1971), one of its interesting results being that changes in λ^+ do not have a strong effect on the magnitude of $(\overline{s_x^2})^{\frac{1}{2}}/\overline{S_x}$.

The observed changes in turbulence structure are qualitatively consistent with a number of explanations for drag reduction which have appeared in the literature. An interpretation based on the notion of anisotropic viscosity particularly appeals to us since it is consistent with our observation that the time-averaged wall stress is related to the time-averaged velocity gradient by Newton's law of viscosity. According to this rheological model, the kinematic viscosity characterizing flow in the axial direction, ν_x, is less than the kinematic viscosity characterizing flow in a plane perpendicular to the x axis, ν_z. Such a hypothesis has been justified by Kobets (1969) and Fortuna (1971), who argue that ellipsoidal molecular coils would orient in a strong shear field with their major axes inclined toward the direction of mean flow. Since the wall eddies are associated with a secondary flow in a plane perpendicular to the direction of mean flow, it would be expected that the magnitude of wavelength λ is influenced more by ν_z than by ν_x. According to these notions much of the observed increase of λ^+ for polymer solutions could be explained by the viscosity used in the evaluation of λ^+ being too small.

This work is sponsored by the National Science Foundation under Grant GK-2813X. The authors are also grateful to Dow Chemical Company for providing the polymer Separan AP-30.

REFERENCES

BAKEWELL, H. P. & LUMLEY, J. L. 1967 *Phys. Fluids*, **10**, 1880.

ECKELMAN, L. D. 1971 The structure of wall turbulence and its relation to eddy transport. Ph.D. thesis in Chemical Engineering, University of Illinois.

FORTUNA, G. 1971 Effect of drag-reducing polymers on flow near a wall. Ph.D. thesis in Chemical Engineering, University of Illinois.

FORTUNA, G. & HANRATTY, T. J. 1971*a* *A.I.Ch.E. Symp. Ser.* **67**, no. **111**, 90.

FORTUNA, G. & HANRATTY, T. J. 1971*b* *Int. J. Heat & Mass Transfer*, **14**, 171.

KLINE, S. J., REYNOLDS, W. C., SCHRAUB, F. A. & RUNSTADLER, P. W. 1967 *J. Fluid Mech.* **30**, 741.

KOBETS, G. F. 1969 *Bionika*, **3**, 72. (Trans. Translation Division Foreign Technology Division WP-AFB, Dayton, Ohio.)

MEYER, W. A. 1966 *A.I.Ch.E. J.* **12**, 522.

MITCHELL, J. E. & HANRATTY, T. J. 1966 *J. Fluid Mech.* **26**, 199.

REISS, L. P. & HANRATTY, T. J. 1963 *A.I.Ch.E. J.* **9**, 154.

SIRKAR, K. K. 1969 Turbulence in the immediate vicinity of a wall and fully developed mass transfer at high Schmidt numbers. Ph.D. thesis in Chemical Engineering, University of Illinois.

SIRKAR, K. K. & HANRATTY, T. J. 1970*a* *J. Fluid Mech.* **44**, 589.

SIRKAR, K. K. & HANRATTY, T. J. 1970*b* *J. Fluid Mech.* **44**, 605.

WELLS, C. S. & SPANGLER, J. G. 1967 *L.T.Y. Research Center Rep., Dallas, Texas*, no. 0.71000/7R-4.